HW 8 SOLUTIONS   SECTION 3.1

2 $\vec{x} = \begin{pmatrix} 2 \\ 3i \\ 1-i \end{pmatrix}$ $\vec{y} = \begin{pmatrix} -1+i \\ 2 \\ 3-i \end{pmatrix}$

(a) $x^T y = (2)(-1+i) + (3i)(2) + (1-i)(3-i)$

$= -2 + 2i + 6i + 3 - 4i - 1$

$= \boxed{4i}$

(b) $y^T y = (-1+i)(-1+i) + (2)(2) + (3-i)(3-i)$

$= 1 - 2i - 1 + 4 + 9 - 6i - 1$

$= \boxed{12 - 8i}$

(c) $\langle x, y \rangle = x^T \bar{y}$ ($\bar{y}$ IS COMPLEX CONJUGATE OF y)

$= (2)(-1-i) + (3i)(2) + (1-i)(3+i)$

$= -2 - 2i + 6i + 3 - 2i + 1$

$= \boxed{2i + 2}$

(d) $\langle y, y \rangle = y^T \bar{y}$

$= (-1+i)(-1-i) + (2)(2) + (3-i)(3+i)$

$= 1 + 1 + 4 + 9 + 1$

$= \boxed{16}$

(3) $A(t) = \begin{pmatrix} e^{t} & 2e^{-t} & e^{2t} \\ 2e^{t} & e^{-t} & -e^{2t} \\ -e^{t} & 3e^{-t} & 2e^{2t} \end{pmatrix}$, $B(t) = \begin{pmatrix} 2e^{t} & e^{-t} & 3e^{2t} \\ -e^{t} & 2e^{-t} & e^{2t} \\ 3e^{t} & -e^{-t} & -e^{2t} \end{pmatrix}$

(a)

$$A + 3B = A + \begin{pmatrix} 6e^{t} & 3e^{-t} & 9e^{2t} \\ -3e^{t} & 6e^{-t} & 3e^{2t} \\ 9e^{t} & -3e^{-t} & -3e^{2t} \end{pmatrix}$$

$$\boxed{= \begin{pmatrix} 7e^{t} & 5e^{-t} & 10e^{2t} \\ -e^{t} & 7e^{-t} & 2e^{2t} \\ 8e^{t} & 0 & -e^{2t} \end{pmatrix}}$$

(b)

$$AB = \begin{pmatrix} e^{t} & 2e^{-t} & e^{2t} \\ 2e^{t} & e^{-t} & -e^{2t} \\ -e^{t} & 3e^{-t} & 2e^{2t} \end{pmatrix} \begin{pmatrix} 2e^{t} & e^{-t} & 3e^{2t} \\ -e^{t} & 2e^{-t} & e^{2t} \\ 3e^{t} & -e^{-t} & -e^{2t} \end{pmatrix}$$

$$\boxed{= \begin{pmatrix} 2e^{2t} - 2 + 3e^{3t} & 1 + 4e^{-2t} - e^{t} & 3e^{3t} + 2e^{t} - e^{4t} \\ 4e^{2t} - 1 - 3e^{3t} & 2 + 2e^{-2t} + e^{t} & 6e^{3t} + e^{t} + e^{4t} \\ -2e^{2t} - 3 + 6e^{3t} & -1 + 6e^{-2t} - 2e^{t} & -3e^{3t} + 3e^{t} - 2e^{4t} \end{pmatrix}}$$

(c)

$$\frac{dA}{dt} = \boxed{\begin{pmatrix} e^{t} & -2e^{-t} & 2e^{2t} \\ 2e^{t} & -e^{-t} & -2e^{2t} \\ -e^{t} & -3e^{-t} & 4e^{2t} \end{pmatrix}}$$

SECTION 3.2

(1) $\vec{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\vec{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\vec{x}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$ ROW REDUCE A:

$$\xrightarrow{r_2 = r_2 - r_1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{r_3 = r_3 - r_2} \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}}_{\text{CALL THIS } \tilde{A}}$$

THE ONLY SOL$^{\underline{n}}$ TO $\tilde{A}c = 0$ IS $\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\Rightarrow$ COLUMNS OF $\tilde{A}$ LINEARLY INDEPENDENT

$\Rightarrow$ $\boxed{\text{COLUMNS OF } A \text{ ARE LINEARLY INDEPENDENT}}$

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(4) $$A = \begin{pmatrix} 2 & -1 & 3 \\ 2 & 3 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$; $\text{DET}(A) = 2 \cdot \text{DET}\begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix} - (-1) \cdot \text{DET}\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$

$+ 3\,\text{DET}\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$

$= -6 - 3 - 9 = \boxed{-18}$

(5) $A = \begin{pmatrix} 1 & 3 & -2 \\ 4 & 0 & -1 \\ -1 & 1 & 1 \end{pmatrix}$

$$\text{DET}(A) = 1 \cdot \text{DET}\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} - 3 \cdot \text{DET}\begin{pmatrix} 4 & -1 \\ -1 & 1 \end{pmatrix} + (-2) \cdot \text{DET}\begin{pmatrix} 4 & 0 \\ -1 & 1 \end{pmatrix}$$

$$= 1 - 9 - 8$$

$$= \boxed{-16}$$

SECTION 3.3

(1) $u'' + \frac{1}{2}u' + 2u = 3\sin(t)$ $\quad 2^{nd}$ ORDER SYSTEM OF 1 EQ^{n}

LET $v = u' \Rightarrow v' = u''$

THEN

$$v' + \tfrac{1}{2}v + 2u = 3\sin(t)$$

$$\left.\begin{aligned} \Rightarrow \quad v' &= -\tfrac{1}{2}v - 2u + 3\sin(t) \\ u' &= v \end{aligned}\right] \quad 1^{st} \text{ ORDER SYSTEM OF 2 EQ}^{n}\text{S.}$$

$$\boxed{\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ 3\sin t \end{pmatrix}}$$

(2) $u^{(4)} - u = 0 \iff \dfrac{d^4 u}{dt^4} - u = 0$

LET $v = u' \Rightarrow v' = u''$

$w = v' \Rightarrow w' = v'' = u'''$

$x = w' \Rightarrow x' = w'' = u^{(4)}$

$\Rightarrow x' - u = 0$

$x' = u$

$$\Rightarrow \boxed{\frac{d}{dt}\begin{pmatrix} u \\ v \\ w \\ x \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \\ x \end{pmatrix}}$$